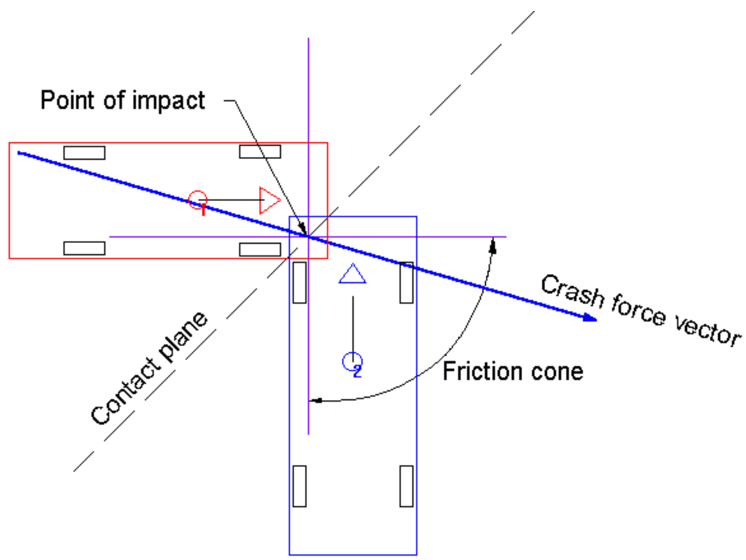
Collision Model Essentials

1.0 Impulse-Restitution Impact Model



1.01 General

The default impact model in PC-Crash is a momentum-based 2 or 3 dimensional model that relies on restitution rather than vehicle crush or stiffness coefficients. This model assumes an exchange of the impact forces within an infinitely small time step at a single point, herein called "impulse point". Instead of resolving the impact forces over time, only the integral of the force-time curve (the impulse) is considered. This model, which was described first by Kudlich[i] and Slibar[ii], contains the means to calculate "full impacts" (impacts in which a common velocity is reached by the contacting areas of the two vehicles) and "sliding impacts" (impacts where no common velocity is reached, commonly called sideswipe impacts).

The impact model allows the calculation of the post-impact parameters after the definition of the pre-impact phase (speeds and positions).

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As defined by Newton the impact can be divided into two phases: the "compression" phase and the "restitution" phase. For a full impact, at the end of the compression phase the velocities of both vehicles at the impulse point are identical.

Due to elasticity of the vehicle structures, the two vehicles will separate again. The coefficient of restitution is defined as ratio between restitution impulse and compression impulse.

$$\varepsilon = \frac{S_R}{S_C} \quad (1.)$$

The total impulse is calculated from

$$S = S_C + S_R = S_C(1+\varepsilon) \quad (2.)$$

For simplicity, the impact model is only derived here in 2D. In PC-Crash both 2D and 3D impact models are available.

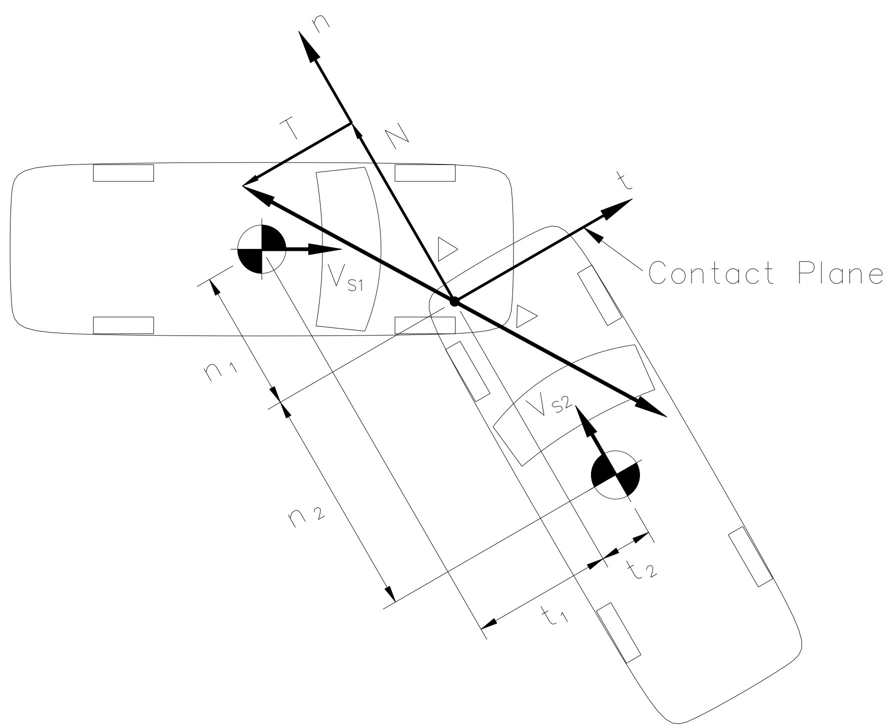

Figure 4.1: impact configuration.

As can be seen in Figure 4.1 a local coordinate system, which originates at the impulse point, is defined. The coordinates are separated into components that are tangential and normal to the defined contact plane between the vehicles. The tangential and normal components of the velocity of each vehicle (only Vehicle 1 equations are shown here) at the impulse point are, respectively:

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$$V_{T1} = V_{Tcg1} - \omega_{z1} n_1 \tag{3.}$$

$$V_{N1} = V_{Ncg1} + \omega_{z1} t_1 \tag{4.}$$

where:

V_{Tcg1} = tangential velocity component of Vehicle 1's center of gravity

V_{Ncg1} = normal velocity component of Vehicle 1's center of gravity

ω_{z1} = yaw velocity of vehicle 1

n_1 = normal component of the center of gravity to impulse point distance

t_1 = tangential component of the center of gravity to impulse point distance

The components of the relative velocity between the two vehicles at the impulse point are:

$$V_T = V_{T1} - V_{T2} \tag{5.}$$

$$V_N = V_{N1} - V_{N2} \tag{6.}$$

In addition the balance of momentum for both vehicles is:

$$m_1(V'_{Tcg1} - V_{Tcg1}) = T \tag{7.}$$

$$m_1(V'_{Ncg1} - V_{Ncg1}) = N \tag{8.}$$

$$m_2(V'_{Tcg2} - V_{Tcg2}) = -T \tag{9.}$$

$$m_2(V'_{Ncg2} - V_{Ncg2}) = -N \tag{10.}$$

The balance of angular momentum is:

$$I_{z1}(\omega'_{z1} - \omega_{z1}) = -T\,n_1 + N\,t_1 \tag{11.}$$

$$I_{z2}(\omega'_{z2} - \omega_{z2}) = T\,n_2 - N\,t_2 \tag{12.}$$

When combining these equations the change of the relative velocity for both vehicles at the impulse point can be calculated from:

$$V'_T = V_T + c_1 T - c_3 N \tag{13.}$$

$$V'_N = V_N - c_3 T + c_2 N \tag{14.}$$

where

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$$c_1 = \frac{1}{m_1} + \frac{1}{m_2} + \frac{n_1^2}{I_{1z}} + \frac{n_2^2}{I_{2z}} \quad (15.)$$

$$c_2 = \frac{1}{m_1} + \frac{1}{m_2} + \frac{t_1^2}{I_{1z}} + \frac{t_2^2}{I_{2z}} \quad (16.)$$

$$c_3 = \frac{t_1 n_1}{I_{1z}} + \frac{t_2 n_2}{I_{2z}} \quad (17.)$$

To be able to solve these equations and to calculate the post-impact velocities and rotations additional definitions have to be made. These definitions vary for the two different kinds of impacts defined by Kudlich and Slibar, as follows.

1.01.1 Full Impact

A full impact is defined as one in which there is no relative movement between the vehicles at the impulse point at the end of the compression phase. In this case two additional assumptions are made:

1. In the compression phase, the exchanged momenta in the tangential and normal directions are:

$$T_C = \frac{V_N c_3 + V_T c_2}{c_3^2 - c_1 c_2} \quad (18.)$$

$$N_C = \frac{V_N c_1 + V_T c_3}{c_3^2 - c_1 c_2} \quad (19.)$$

2. The ratio between compression and restitution impulse is defined by the coefficient of restitution, according to EQ (136).

   The components of the total exchanged momentum can be calculated from:

$$T = T_C\,(1+\varepsilon) \quad (20.)$$

$$N = N_C\,(1+\varepsilon) \quad (21.)$$

These equations are sufficient to calculate all post-impact velocity conditions for both involved vehicles in case of a full impact.

1.01.2 Sliding Impact

In a sliding impact, the two vehicles do not reach a common velocity at the impulse point during the impact. In such a case a contact plane has to be defined, along which the two vehicles slide. The impulse point must lie in this plane. For this case, the following assumptions are made:

1. No relative movement between the vehicles occurs at the impulse point at the end of the compression phase in the direction normal to the contact plane. The normal component of the impulse can, however, be affected by the inter-vehicle friction coefficient (μ), and N_C is now calculated from:

$$N_C = \frac{V_N}{\mu c_3 + c_2} \tag{22.}$$

2. The direction of the momentum transfer is limited by μ, such that:

$$T_C = \mu N_C \tag{23.}$$

3. The ratio between compression and restitution impulse is again defined by the coefficient of restitution according to EQ (136) and T and N can again be calculated from EQ (155) and EQ (156).

The post impact velocity conditions for both vehicles in the impact are calculated from the preceding relations.

1.02 Impact Positions

It is important for a good prediction of the collision phase to define the correct overlapping of the original vehicle outlines when the forces are exchanged.

The best method of doing this is to position the vehicles at the impact point in an overlapped position corresponding to the amount of crush. In PC-Crash this position can also be determined in another way. Crash tests have shown that the time from the first contact of the vehicles to the maximum engagement usually lasts a time period of 30 to 60 milliseconds. Based on this, the correct overlapping of the vehicles is reached by driving the vehicles at the assumed pre-impact velocities from the point of first contact for a distance corresponding to this time period. With the vehicles in this position, the impact calculation is performed.

Finally the position where the impact forces interact at maximum engagement (the impulse point) must be defined by the user. When identifying this point on the screen the local stiffness of the vehicle body and underlying structure should be taken into account. The location of this point can be refined by considering the post-impact trajectories of the two vehicles.

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1.03 Coefficient of Restitution

The coefficient of restitution is an input parameter for PC-Crash. In real vehicle to vehicle collisions, it usually lies in the range between 0.1 and 0.3. Generally, the higher the residual deformation of the vehicles, the lower the coefficient of restitution. Values higher than 0.3 can occur for low approach velocities[iii,iv,v,vi], especially when little or no damage occurs.

The elasticity of an impact can also be defined by specifying the separation velocity. In real vehicle to vehicle collisions, this value is usually around 5 km/h (3 mph), regardless of approach speed. This method cannot be used for sliding impacts, however.

2.0 Derived Quantities

2.01 Deformation Depth

The impact model is not based on crush energy or vehicle stiffness. However, the deformation of each vehicle is estimated as a check against the physical evidence.

The deformation S_i of vehicle *i* is estimated as the distance from the point of impact to the surface of the contact detection outline along the line of action of the collision impulse.

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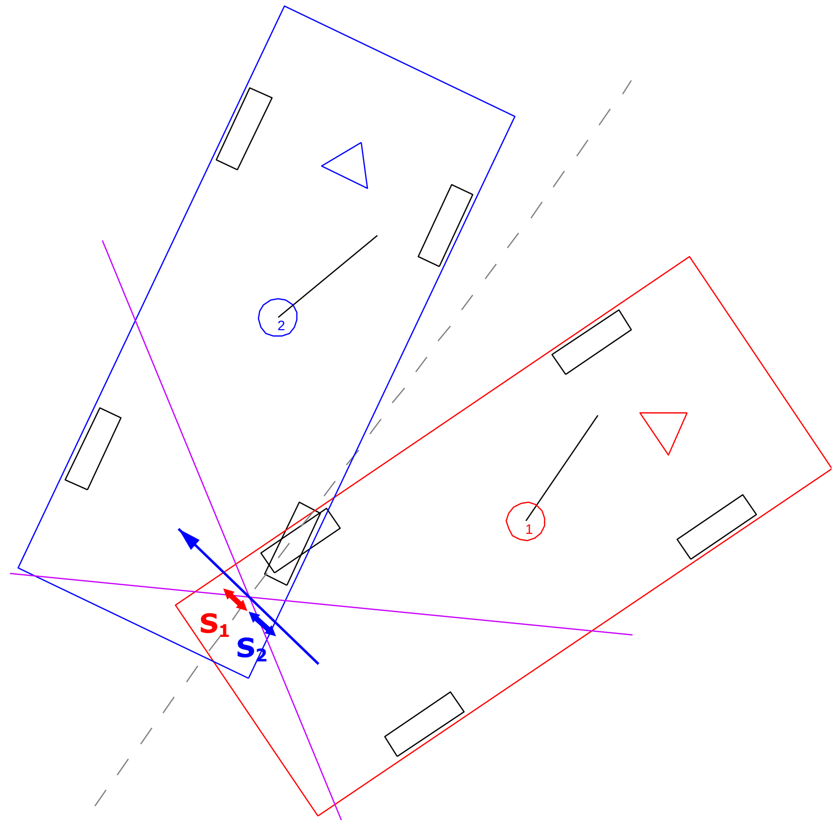

2.02 Deformation Energy

The kinetic energy of the system can be calculated before and after the collision. The difference in kinetic energy is the Deformation Energy E_D.

$$E_D = \sum_i \tfrac{1}{2} m_i \left(v_i^2 - v_i'^2 \right) + \sum_i \tfrac{1}{2} I_i \left(\omega_i^2 - \omega_i'^2 \right) \qquad (24.)$$

2.03 Calculation of EES

After the definition of the vehicle positions at impact and the point of impact, the relative deformation depth of each vehicle is known.

The following are the relationships used in PC-Crash (as defined by Burg[vii]) for deformation energy (EES, or Equivalent Energy Speed):

$$\frac{EES_1}{EES_2} = \sqrt{\frac{m_2}{m_1} \frac{s_{Def1}}{s_{Def2}}} \qquad (25.)$$

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$$EES_2 = \sqrt{\frac{2E_D}{m_2\left(\frac{s_{Def1}}{s_{Def2}}+1\right)}} \qquad (26.)$$

where

m_1, m_2 = Mass of each vehicle

s_{Def1}, s_{Def2} = Crush depth of each vehicle, outer surface to impact point in line with impact force

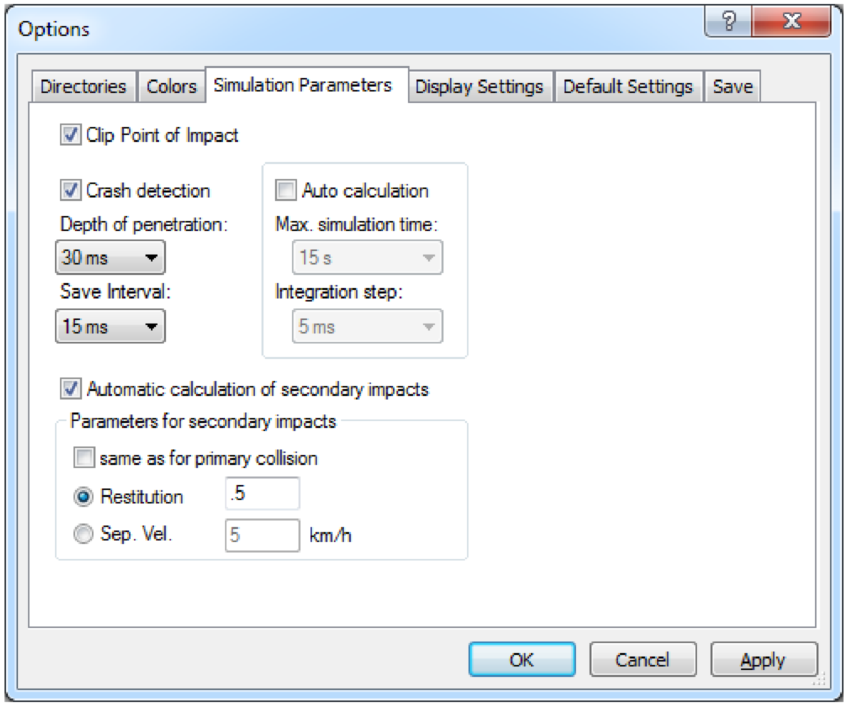
E_D = Energy lost by both vehicles in the collision due to damage.

3.0 Crash Detection & Automatic Calculation

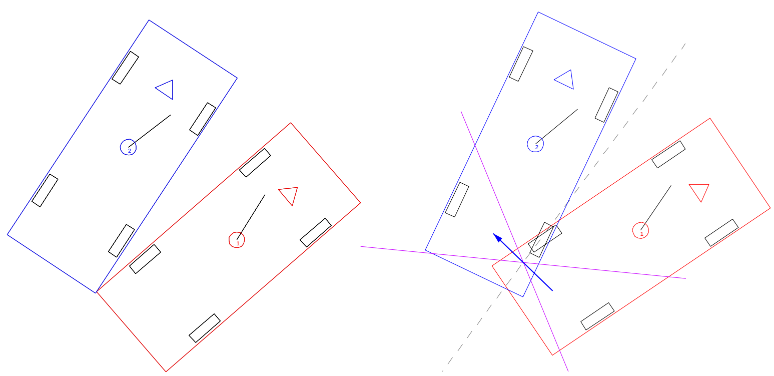
After the initial impact, vehicles may come into contact again, with each other or with other vehicles. It is possible to simulate until secondary impacts occur, and manual specify the impact parameters at each impact, just like the initial impact. This approach is time consuming and error-prone. Therefore, PC-Crash can detect secondary impacts, and can automatically assign parameters to use for the calculation of these impacts.

3.01 Crash Detection

With Crash Detection enabled, vehicle plan view outlines are constantly monitored for overlap. Overlap is quantified by the area of overlap. If overlap increases with time, the vehicles are probably approaching and an impact should occur.



The Depth of penetration is the duration of increasing overlap after which an impact should occur. If Automatic calculation of secondary impacts is selected, PC-Crash will perform an impact calculation at that time. If Automatic calculation of secondary impacts is not selected, then the simulation will stop and the Crash Simulation dialog will open for the user to assign impact parameters.



Start of overlap (left), and impact detected (right).

3.02 Automatic Calculation of Secondary Impacts

When secondary impacts are calculated automatically, the following process is used to determine the impact parameters to use for the calculation:

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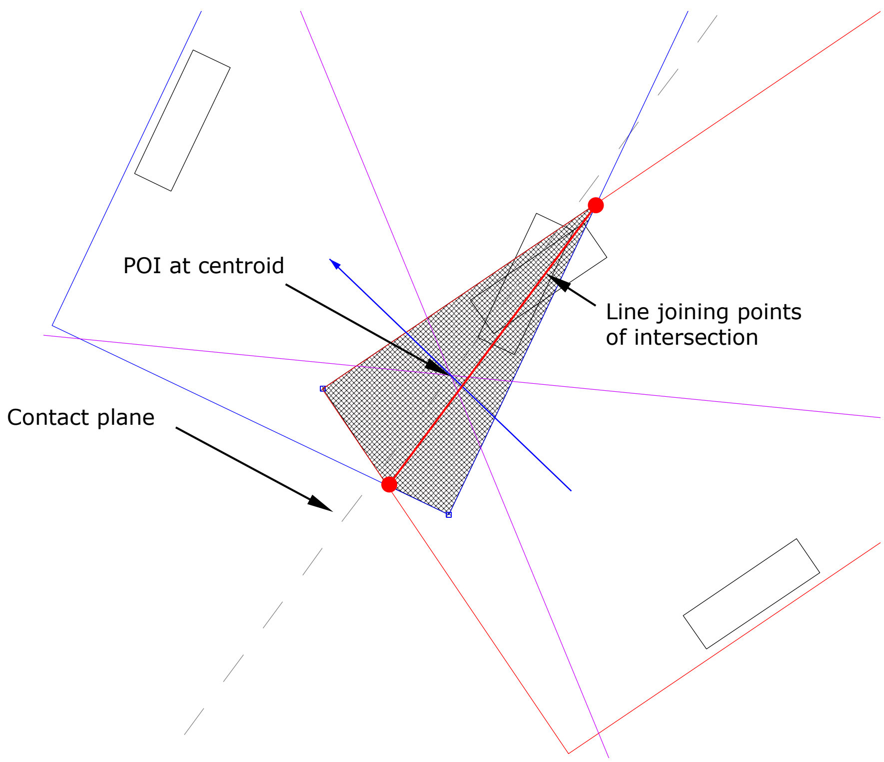
- The X & Y values for the Point of Impact are the centroid of the area of overlap
- The Z value for the Point of Impact is taken from vehicle impact parameters, or the value from the initial impact
- The Contact Plane angle is taken from vehicle impact parameters, or is parallel to the line joining the points of intersection of the overlapping area
- The Contact Plane inclination is taken from vehicle impact parameters, of the value from the initial impact
- The Contact Friction is taken from vehicle impact parameters, or the value from the initial impact
- The Restitution Coefficient is taken from vehicle impact parameters, or the secondary impact value specified, or the value from the initial impact



3.03 Vehicle-Based Impact Parameters

The impact parameters can be specified on a vehicle basis. This means, instead of determining the impact parameters by calculations the parameters specified on this page are used. This is useful for cases with unusual engagement.

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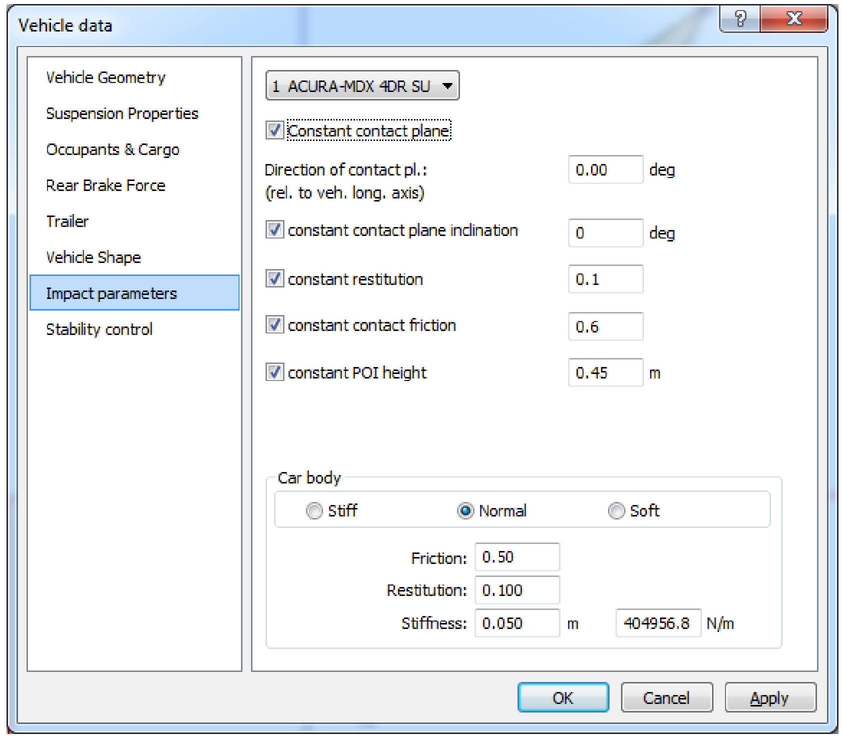


Constant contact plane

The direction of the contact plane (phi in the Crash Simulation window) can be set to a constant value relative to the vehicles longitudinal axis. If this option is turned off, the contact plane will be determined by the points of intersection of the overlapping vehicle plan view outlines.

Constant contact plane inclination

The direction of the contact plane (psi in the Crash Simulation window) can be set to a constant value relative to the horizontal axis.

Constant restitution

For collisions with this vehicle the factor of restitution can be set to a constant value.

Constant contact friction

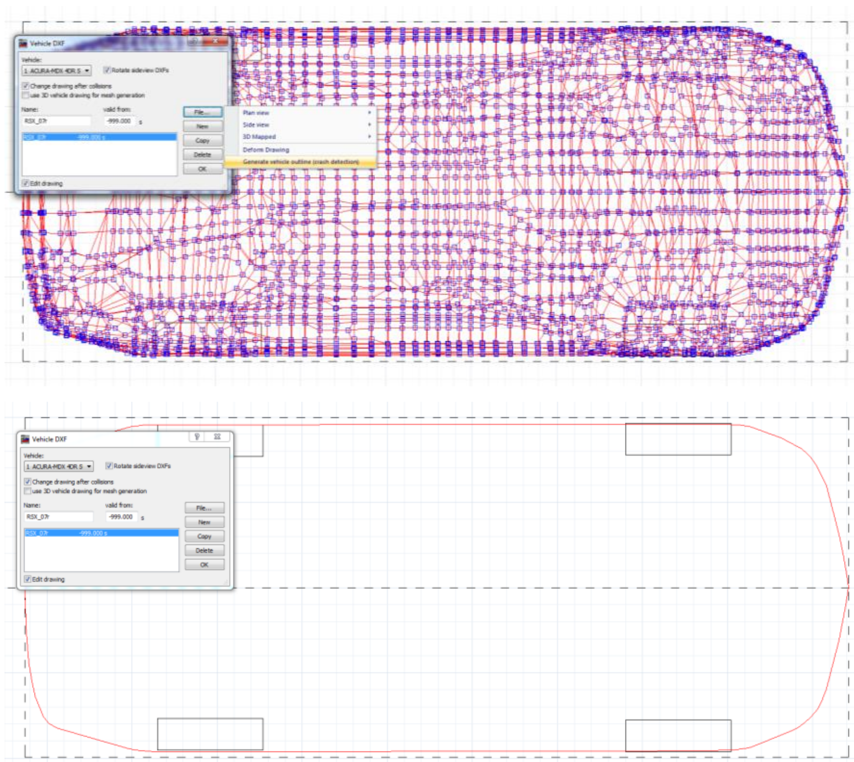
For collisions with this vehicle the friction can be set to a constant value.

Constant POI height

For collisions with this vehicle the Z value of the POI can be set to a constant value.

3.04 Custom Contact Detection Outlines

By default, collision detection is performed using a rectangle based on the length, width, and orientation of the vehicle. A custom outline can be defined in the Vehicle DXF window. If a 2D or 3D vehicle DXF or portion thereof is selected, a contact detection outline can be generated based on the selection. The outline will be convex, and will be bounded by the vehicle length and width. It can be modified after generation through the usual Vehicle DXF editing tools.



DXF selected (top), and Custom outline generated (bottom).

4.0 The Collision Optimizer

The collision optimizer in PC-Crash 3D automatically varies pre-impact parameters to minimize the error between the actual and calculated intermediate and rest positions[viii]. Pre-impact parameters that can be varied by the optimizer are:

- Impact velocity
- Position of impact (POI) in x-y direction
- POI in z direction (3D simulations only)

- Contact plane angle phi (degrees CCW from the global x-axis)
- Pre-impact directions
- Vehicle positions
- Restitution
- Contact plane friction

The contact plane angle psi (used for 3D simulations only) and vehicle pre-impact rotational speeds (if they exist), are not determined automatically with the Optimizer and have to be specified or varied by the user.

4.01 Weighted Total Error

The goal of the optimization process is to minimize the weighted total error, based on the least mean squares method. The weighted total error or quality function *Q* is as follows:

$$Q = \sqrt{\frac{\sum_i (w_i \cdot x_i)^2}{\sum_i w_i^2}} \cdot 100\% \qquad (27.)$$

where

w_i = Weighting (0 to 100%) of each parameter

x_i = Difference between actual and calculated parameter values, as follows:

Positional errors for rest positions,

$$x_{Rest-Position} = \frac{\left|\boldsymbol{P}_{Rest-Sim} - \boldsymbol{P}_{Rest-actual}\right|}{\left|\boldsymbol{P}_{Rest-actual} - \boldsymbol{P}_{impact-actual}\right|} \qquad (28.)$$

Positional errors for intermediate positions,

$$x_{Inter-Position} = \frac{\left|\boldsymbol{P}_{Inter-sim} - \boldsymbol{P}_{Inter-actual}\right|}{\left|\boldsymbol{P}_{Inter-actual} - \boldsymbol{P}_{impact-actual}\right|} \qquad (29.)$$

Heading errors for rest positions,

$$x_{Rest-heading_i} = \frac{\arccos(\boldsymbol{d}_{Rest-actual} \cdot \boldsymbol{d}_{Rest-sim})}{\pi} \qquad (30.)$$

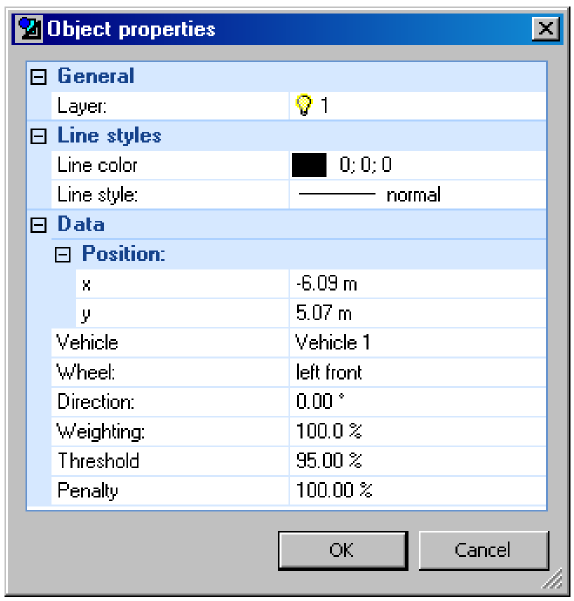
Heading errors for intermediate positions,

$$x_{Inter-heading_i} = \frac{\arccos(\mathbf{d}_{Inter-actual} \cdot \mathbf{d}_{Inter-sim})}{\pi} \qquad (31.)$$

4.02 Define tiremarks for optimizer

Tiremarks can be assigned to a wheel for the Collision Optimzer. The positions of tiremarks are used in the calculation of the Optimizer Error (see below) .

First select menu item **<Impact> Define tiremarks for optimizer** (or select  in the Draw Toolbar) and set a desired point on the tiremark by clicking left mouse button in the working area. Then select this tiremark. By double-click on the tiremark you can change its properties. Multiple tiremarks can be shift-selected to edit their properties all at once.



Line color and line style can be changed.

Vehicle: The vehicle to which the tiremark is assigned.

Wheel: The wheel that made the tiremark (left front, right front, left rear, right rear).

Direction: The direction of the tire mark as it passes through this point. This value has no influence on the calculation. It is for documentation purposes only.

Weighting, Threshold, and Penalty:

The Threshold and Penalty factors allow the user to differentiate between wheel trajectories and tire marks. The contribution to the Optimizer Error is calculated as follows:

1) The closest point on the specified wheel to the tiremark is calculated.
2) The error in this tiremark position, E_{pos}, is calculated as

$$E_{Pos} = \left\| P_{TireSim} - P_{TireOpt} \right\|$$

3) The friction used at this point on the wheel trajectory is compared to the Threshold. The application of the penalty depends on the percentage of available friction *F* used by the wheel at that point, the Threshold *T*, and the sign of the Penalty *k*, as follows:

$$x_i = \begin{cases} \|k\| E_{Pos} & k > 0 \,\&\, f < T \quad or \quad k < 0 \,\&\, f \geq T \\ E_{Pos} & k > 0 \,\&\, f \geq T \quad or \quad k < 0 \,\&\, f < T \end{cases}$$

4) If the TM-Easy tire model is used, and the Threshold T < 0, then the tire slip is compared to the Threshold rather than the utilized friction.

4.03 Optimization Methods

Two optimization methods are included in PC-Crash:

4.03.1 Genetic Method

The genetic or evolutionary approach is a very general approach for solving multidimensional optimization problems. This method can be used to optimize subsets of the input parameters or all of them at the same time. The parameters are divided into subsets that are optimized iteratively. This is very useful for the accident reconstruction process.

Starting with initial values (the first generation), the Genetic Method varies the start values leading to a number of pre-impact configurations. A simulation is performed for all configurations and the quality function is evaluated. The parameter set of the second generation which leads to the best results (the lowest quality function values), is used as an initial value in the next step. Different step widths are used in the variation of the different pre-impact parameters. The sequence of variation and evaluation is performed until no further improvements can be achieved and a minimum value for the quality function has been found.

In a further process the step width for the variation is decreased, and a more accurate optimization around the minimum quality function value is performed using the same procedure. Depending on the initial values for the first generation the Genetic Method can solve the problem in a few steps.

The Genetic Method is also very efficient when doing a parameter variation for an existing solution. This is very useful for examining the influence of each different pre-impact parameter for a particular case.

The Genetic Method is practical for all input parameters in PC-Crash and has good numerical stability and performance. Also, the Genetic Method does not tend to stop at local minima.

4.03.2 Monte Carlo Method

The Monte Carlo algorithm performs 100 calculations each time an optimization is made. For each calculation, random values are used for all selected optimization parameters. This avoids situations where the other algorithms could stop at a local minimum. It also gives a good indication of the expected ranges of velocities and other parameters.

The ranges of the random values are:

- **Velocity** - the Vmin and Vmax values in the Properties dialog box are used. The default range for each vehicle is 0 to 300 km/h (186 mph).
- **POI in x-y direction** –+/- 0.5m (1.6 feet)
- **POI in z direction** - +/-0.25m (0.8 feet).
- **Contact plane angle** - +/-15°
- **Pre-impact directions** - +/-15°
- **Vehicle positions** - +/-0.1m (0.3 feet)
- **Coefficient of restitution** - +/-0.25
- **Contact plane friction** - +/-0.25.

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[i] Kudlich, H.,
"Beitrag zur Mechanik des Kraftfahreug-Verkehrsunfalls", Dissertation TU-Wien, 1966.

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"Die mechanischen Grundsätze des Stoßvorganges freier und geführter Körper und ihre Anwendung auf den Stoßvorgang von Fahrzeugen", Archiv für Unfallforschung, 2. Jg., H. 1, 1966, 31ff.

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"Data and Methods for Estimating the Severity of Minor Impacts", SAE 950352

[iv] King, David J., Siegmund, Gunter P., Bailey, Mark N., "Automobile Bumper Behavior in Low-Speed Impacts", SAE 930211.

[v] Siegmund, G.P., Bailey, M.N., and King, D.J., "Characteristics of Specific Automobile Bumpers in Low-Velocity Impacts", SAE 940916.

[vi] Siegmund, Gunter P, King, David J, and Montgomery, Darcy T., "Using Barrier Impact Data to Determine Speed Change in Aligned, Low Speed Vehicle-to-Vehicle Collisions" SAE 960887.

[viii] Moser, A.and Steffan, H., "Automatic Optimization of Pre-Impact Parameters Using Post-Impact Trajectories and Rest Positions", SAE 980373.